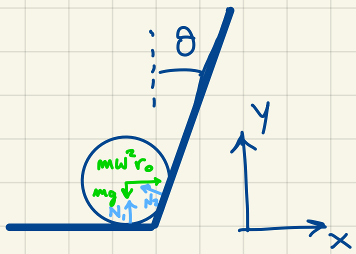
ESERCIZIO 1



a) Sistema di riferimento rotante a ω

Condizione di equilibrio:

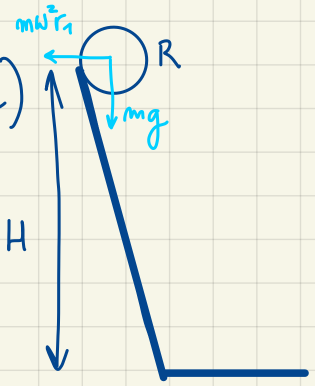
$x \longrightarrow m\omega^2 r_0 - N_2 \cos\theta = 0 \Rightarrow \boxed{N_2 = \frac{m\omega^2 r_0}{\cos\theta}}$

$y \longrightarrow N_1 - mg + N_2 \sin\theta = 0 \Rightarrow \boxed{N_1 = mg - m\omega^2 r_0 \, tg\theta}$

b) Il tubo inizia a risalire quando $N_1 = 0$

$mg = m\omega_c^2 r_0 \, tg\theta \longrightarrow \boxed{\omega_c = \sqrt{g / r_0 tg\theta}}$

c)



Il tubo parte da fermo nel SR rotante a $\frac{\omega_c}{2}$

$v(0) = 0 \ m/s$

Poi inizia a scendere di rotolamento puro

$v(y,t) = R\omega(y,t) \qquad I = mR^2$

$E_K(y,t) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2\left[1 + \frac{I}{mR^2}\right] = mv^2$

Solo forze conservative fanno lavoro, quindi Em si conserva

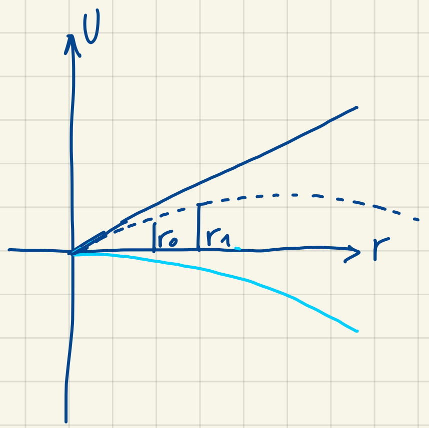
$mgH - \frac{1}{2}m\omega^2 r_1^2 = mv(H)^2 - \frac{1}{2}m\omega^2 r_0^2$

$\omega = \frac{\omega_c}{2}$

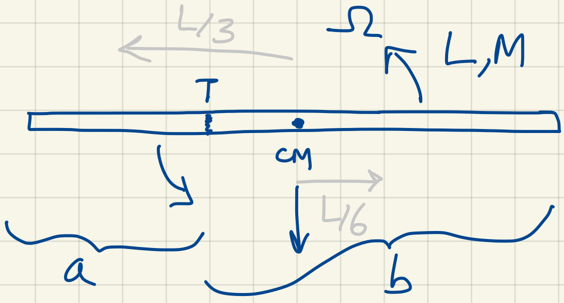
$\frac{r_1 - r_0}{\sin\theta} = \frac{H}{\cos\theta}$

$r_1 = r_0 + H \, tg\theta$

$$\boxed{v(H) = \sqrt{gH - \frac{\omega_c^2}{8}\left(2r_0 H tg\theta + H^2 tg^2\theta\right)}}$$



ESERCIZIO 2



a) $E_K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \Omega^2$

$I_{cm} = \frac{1}{12} M L^2 \qquad v_{cm} = g t$

$$\boxed{E_K = \frac{1}{2} M g^2 t^2 + \frac{1}{24} M L^2 \Omega^2} = \overset{\text{TRASL.}}{E_K \text{ del CM}} + \overset{\text{ROT}}{E_K \text{ rispetto al CM}}$$

b) L_{cm} si conserva perché la rottura è causata da forze interne

Prima : $L_{cm} = I_{cm} \Omega = \frac{1}{12} M L^2 \Omega$

Dopo : $L^{(CM)} = \sum_i \left(\vec{r}_i \times m_i \vec{v}_i + I_i^{(CM)} \omega_i \right) = \frac{L}{3} \frac{M}{3} \frac{L}{3} \Omega + I_a \omega_a + \frac{L}{6} \frac{2M}{3} \frac{L}{6} \Omega + I_b \omega_b$

$I_a^{(CM)} = \frac{1}{12} \frac{M}{3} \frac{L^2}{9} \qquad I_b^{(CM)} = \frac{1}{12} \frac{2M}{3} \frac{4L^2}{9}$

$$\frac{1}{12} M L^2 \Omega = \frac{1}{27} M L^2 \Omega + \frac{1}{2} \frac{1}{27} M L^2 \Omega + \frac{1}{12} \frac{1}{27} M L^2 \omega_a + \frac{1}{12} \frac{8}{27} M L^2 \omega_b$$

$$27\Omega - 12\Omega - 6\Omega = \omega_a + 8\omega_b = 9\Omega \longrightarrow \omega_b = \frac{9}{8}\Omega - \frac{\omega_a}{8}$$

$\omega_a = 2\Omega$

$$\boxed{\omega_b = \frac{7}{8}\Omega}$$

$\Rightarrow$ Dopo la rottura, rispetto al CM $\rightarrow E_K^{(CM)} = \frac{1}{2} \frac{M}{3} \left(\frac{L}{3} \Omega \right)^2 + \frac{1}{2} I_a \omega_a^2 + \frac{1}{2} \frac{2M}{3} \left(\frac{L}{6} \Omega \right)^2 + \frac{1}{2} I_b \omega_b^2$

$$E_K = E_K^{(CM)} + \frac{1}{2} M g^2 t^2$$

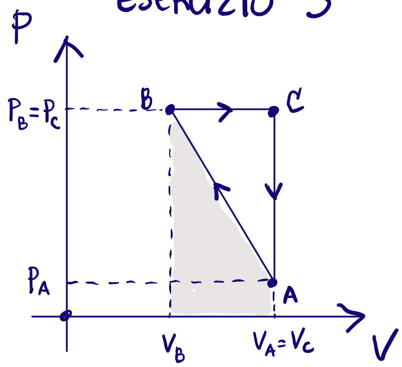
c) $\boxed{W_{tarlo} = \Delta E_K} =$

$$= \frac{1}{2} \frac{M}{3} \frac{L^2 \Omega^2}{9} + \frac{1}{2} \frac{1}{12} \frac{M}{3} \frac{L^2}{9} \underbrace{\omega_a^2}_{4\Omega^2} + \frac{M}{3} \frac{L^2 \Omega^2}{36} + \frac{1}{2} \frac{1}{12} \frac{1}{3} \frac{4}{9} M L^2 \underbrace{\omega_b^2}_{\frac{49}{64}\Omega^2} - \frac{1}{24} M L^2 \Omega^2 =$$

$$= \frac{1}{12} M L^2 \Omega^2 \left[\frac{1}{3} + \frac{2}{27} + \frac{1}{9} + \frac{\cancel{2}}{27} \frac{49}{\cancel{64}_{32}} - \frac{1}{2} \right] = \frac{1}{2} M L^2 \Omega^2 \, \frac{188 + 64 + 96 + 49 - 432}{27 \cdot 32} =$$

$$= \frac{35}{864} \frac{1}{2} M L^2 \Omega^2 = 0.02\, M L^2 \Omega^2 \sim \boxed{11.7 \text{ mJ}}$$

ESERCIZIO 3



$P_B = P_C = 3P_A$ $\qquad$ $n = 1$

$V_A = V_C = 2V_B$

Andamento $P(V)$ nel tratto AB:

$$P = \alpha V + \beta \;, \quad \begin{matrix} P_A = \alpha V_A + \beta \\ P_B = \alpha V_B + \beta \end{matrix} \Rightarrow$$

$$\Rightarrow \alpha = \frac{P_A - P_B}{V_A - V_B} = \frac{P_A(1 - P_B/P_A)}{V_A(1 - V_B/V_A)} = -4\frac{P_A}{V_A}$$

$$\beta = P_A - \alpha V_A = P_A + 4P_A = 5P_A$$

$$\Rightarrow P = -4P_A\frac{V}{V_A} + 5P_A = P_A\left(-4\frac{V}{V_A} + 5\right)$$

$$\underline{W_{AB}} = -\frac{(P_A + P_B)(V_A - V_B)}{2} = -\frac{4P_A \cdot V_A/2}{2} = -P_A V_A = \underline{-RT_A} \qquad \left(\text{area del trapezio}\right)$$

$$\Delta U_{AB} = C_V(T_B - T_A)\,; \qquad T_B = \frac{P_B V_B}{R} = \frac{3P_A \cdot V_A/2}{R} = \frac{3}{2}T_A$$

$$\Rightarrow \underline{\Delta U_{AB}} = C_V\frac{T_A}{2} = \underline{\frac{3}{4}RT_A} \qquad \Rightarrow \underline{Q_{AB}} = \Delta U_{AB} + W_{AB} = \underline{-\frac{1}{4}RT_A}$$

$$\underline{W_{BC}} = P_B(V_C - V_B) = 3P_A \cdot V_A/2 = \underline{\frac{3}{2}RT_A}$$

$$Q_{BC} = C_p(T_C - T_B)\,; \qquad T_C = \frac{P_C V_C}{R} = \frac{3P_A \cdot V_A}{R} = 3T_A$$

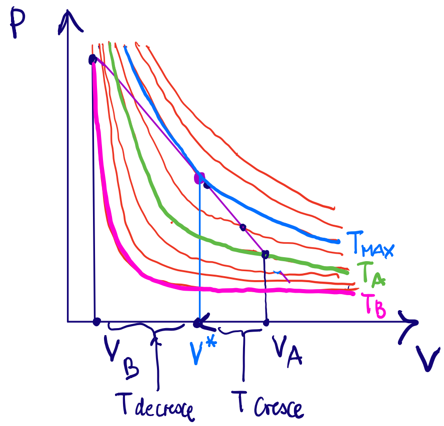
$$\Rightarrow \underline{Q_{BC}} = C_p\left(3T_A - \frac{3}{2}T_A\right) = \frac{3}{2}C_p T_A = \underline{\frac{15}{4}RT_A}$$

$$\underline{\Delta U_{BC}} = C_V(T_C - T_B) = \frac{3}{2}C_V T_A = \underline{\frac{9}{4}RT_A}$$

$$\underline{W_{CA} = 0}$$

$$\underline{\Delta U_{CA}} = Q_{CA} = C_V(T_A - T_C) = C_V(T_A - 3T_A) = \underline{-3RT_A}$$

$$\underline{W_{TOT}} = \underline{Q_{TOT}} = -RT_A + \frac{3}{2}RT_A = \underline{\frac{RT_A}{2}}$$



Nel tratto lineare AB si hanno due segmenti con cessione e assorbimento di calore.

Conviene calcolare $\frac{dQ}{dV}$:

$$\delta Q = c_V dT + P dV = \left(c_V \frac{dT}{dV} + P\right) dV$$

e $T = \frac{PV}{R} = \frac{(\alpha V + \beta)V}{R}$

$$\Rightarrow \frac{dT}{dV} = \frac{1}{R}(2\alpha V + \beta)$$

$$\Rightarrow \delta Q = \left[\frac{c_V}{R}(2\alpha V + \beta) + P\right] dV = \left[\frac{3}{2}(2\alpha V + \beta) + \alpha V + \beta\right] dV = \left[4\alpha V + \frac{5}{2}\beta\right] dV$$

$$\Rightarrow \frac{\delta Q}{dV} = \left(4\alpha V + \frac{5}{2}\beta\right) = -16 P_A \frac{V}{V_A} + \frac{25}{2} P_A = P_A\left[-16\frac{V}{V_A} + \frac{25}{2}\right]$$

Si vede che $\left(\frac{\delta Q}{dV}\right) \gtrless 0$ per $V \lessgtr \frac{25}{32} V_A = V^*$

Quindi, a partire da V_A, quando $\begin{cases} V^* < V < V_A & Q \text{ è assorbito dal gas} \\ V_B < V < V^* & Q \text{ è ceduto al gas} \end{cases}$

Calcolo di questi due calori:

$$Q_{A\to *} = \int_{V_A}^{V^*} \frac{dQ}{dV} dV = P_A \int_{V_A}^{V^*} \left(-16\frac{V}{V_A} + \frac{25}{2}\right) dV = P_A\left[-\frac{8V^2}{V_A} + \frac{25}{2}V\right]_{V_A}^{V^*} =$$

$$= P_A\left[-\frac{8}{V_A}\left(\frac{25}{32}\right)^2 V_A^2 + \frac{25}{2}\cdot\frac{25}{32}V_A + \frac{8V_A^2}{V_A} - \frac{25}{2}V_A\right] = P_A V_A\left[-\frac{25^2\cdot 8}{32^2} + \frac{25^2}{2\cdot 32} + 8 - \frac{25}{2}\right] = \frac{49}{128} RT_A$$

$$Q_{*\to B} = \int_{V^*}^{V_B = V_A/2} \frac{dQ}{dV} = P_A\left[-\frac{8V^2}{V_A} + \frac{25}{2}V\right]_{V^*}^{V_A/2} = P_A\left[-\frac{8}{V_A}\cdot\frac{V_A^2}{4} + \frac{25}{4}V_A + \frac{8}{V_A}\cdot\frac{25^2}{32^2}V_A^2 - \frac{25}{2}\cdot\frac{25 V_A}{32}\right] =$$

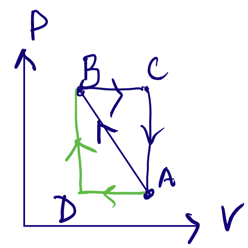
$$= P_A V_A\left[-2 + \frac{25}{4} + \frac{25^2}{32}\left[\frac{8}{32} - \frac{1}{2}\right]\right] = -\frac{81}{128} RT_A$$

[NB: $Q_{A\to *} + Q_{*\to B} = \left(\frac{49}{128} - \frac{81}{128}\right) RT_A = -\frac{1}{4} RT_A = Q_{AB}$]

Calcolo dell'efficienza: $\eta = \dfrac{W_{TOT}}{Q_{IN}}$, $Q_{IN} = Q_{BC} + Q_{A\to *} =$

$$= \left(\frac{15}{4} + \frac{49}{128}\right) RT_A = \frac{529}{128} RT_A$$

$$\eta = \frac{RT_A}{2} \cdot \frac{128}{529\, RT_A} = \frac{64}{529} \simeq 0.12$$



Calcolo dell'entropia:

$$\Delta S^{gas}_{AB} = \Delta S^{gas}_{AD} + \Delta S^{gas}_{DB} =$$

$$= c_p \ln \frac{T_D}{T_A} + c_v \ln \frac{T_B}{T_D}$$

$$RT_D = P_A V_B = \frac{P_A V_A}{2} = \frac{RT_A}{2} \Rightarrow T_D = \frac{T_A}{2}$$

$$RT_B = P_B V_B = 3P_A \frac{V_A}{2} = \frac{3}{2} RT_A \Rightarrow T_B = \frac{3}{2} T_A$$

$$RT_C = P_C V_C = 3 P_A V_A \Rightarrow T_C = 3 T_A$$

$$\Delta S^{gas}_{AB} = c_p \cdot \ln \frac{1}{2} + c_v \ln 3 =$$

$$= R\left(\frac{5}{2} \ln \frac{1}{2} + \frac{3}{2} \ln 3\right) \simeq -0.71 \text{ J/K}$$

$$\Delta S^{gas}_{BC} = c_p \ln \frac{T_C}{T_B} = \frac{5}{2} R \ln 2 \simeq 14.4 \text{ J/K}$$

$$\Delta S^{gas}_{CA} = c_v \ln \frac{T_A}{T_C} = \frac{3}{2} R \ln \frac{1}{3} \simeq -13.69 \text{ J/K}$$