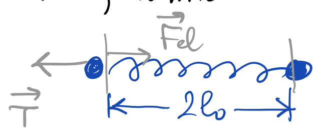
1 a. Condizione di equilibrio statico : $\vec{T} + \vec{F}_{el} = \vec{0}$



$$\downarrow$$
$$T = F_{el} = k(2\ell_0 - \ell_0) = k\ell_0$$

b. Equazione di moto nel riferimento in rotazione con velocità Ω

$$\mu \ddot{\ell} = -k(\ell - \ell_0) + \mu \Omega^2 r \qquad , \quad \mu = \frac{m}{2}$$

condizione di distacco per $\ell = 2\ell_0$, $\Omega = \Omega_1$:

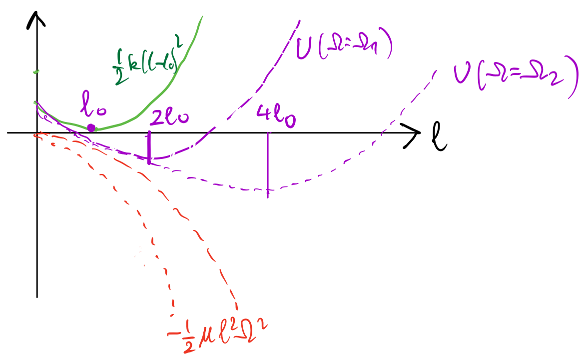
$$-k\ell_0 + \mu\Omega_1^2 \cdot 2\ell_0 = 0 \iff \Omega_1 = \sqrt{\frac{k}{2\mu}} = \sqrt{\frac{k}{m}}$$

c. condizione per $\ell = 4\ell_0$, $\Omega = \Omega_2$

$$-3k\ell_0 + \mu\Omega_2^2 \cdot 4\ell_0 = 0 \iff \Omega_2 = \sqrt{\frac{3k}{4\mu}} = \sqrt{\frac{3k}{2m}}$$

d. $L_O = \mu v_{rel} \cdot \ell = \mu \ell^2 \Omega$

e. $U = \frac{1}{2}k(\ell - \ell_0)^2 - 2 \cdot \frac{1}{2} m \Omega^2 \left(\frac{\ell}{2}\right)^2 = \frac{1}{2}k(\ell - \ell_0)^2 - \frac{1}{4} m \ell^2 \Omega^2$



f.

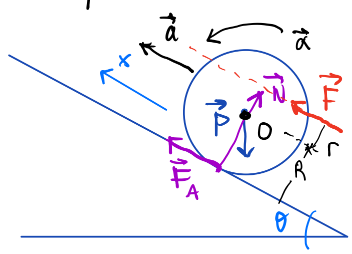
$$\frac{dU}{d\ell} = 0 \iff k(\ell_{eq} - \ell_0) = \frac{1}{2} m \ell_{eq} \Omega^2 \iff \ell_{eq} = \frac{k\ell_0}{k - \frac{1}{2}m\Omega^2}$$

$$F(\ell) = -\frac{dU}{d\ell} = -k(\ell - \ell_0) + \frac{1}{2}m\ell\Omega^2 = -\ell\left(k - \frac{m\Omega^2}{2}\right) + k\ell_0$$

$$\Rightarrow \omega_{osc}^2 = \frac{k - m\Omega^2/2}{\mu} \Rightarrow \omega_{osc} = \sqrt{\frac{k}{\mu} - \Omega^2}$$

g. Si conserva il momento angolare, Ω e ℓ oscillano in modo che $\Omega \ell^2 = \text{cost.}$

2. Equazioni cardinali



$\vec F + \vec F_A + \vec P + \vec N = M\vec a$

$\vec r \times \vec F + \vec R \times \vec F_A = \vec \tau_O = I_{CM}\vec \alpha$

Proiezioni
$$F + F_A - Mg\sin\vartheta = Ma$$
$$rF - RF_A = I_{CM}\alpha = I_{CM}\, a/R$$
puro rotolamento

a. È un moto uniformemente accelerato del CM: $x = \frac{1}{2}at^2$, $v = at$

$$\Rightarrow d = \frac{1}{2}a t_F^2,\quad v_F = a t_F \Rightarrow a = \frac{2d}{t_F^2} = \frac{2d}{v_F^2}\cdot a^2 \Rightarrow a = \frac{v_F^2}{2d}$$

b. $I_{CM} = 2I_{Base} + I_{LATERALE} = 2\cdot\frac{M_{BASE}R^2}{2} + M_{LAT}\cdot R^2 = (M_{BASE}+M_{LAT})R^2$;

densità superficiale (kg/m²) $\delta = \frac{M}{Area} = \frac{M}{2\pi R^2 + 2\pi R\cdot H} \Rightarrow M_{BASE} = \delta\cdot\pi R^2 = \frac{M}{2}\cdot\frac{R}{R+H}$

$M_{LAT} = \delta\cdot 2\pi R H = M\cdot\frac{H}{R+H}$

$$\Rightarrow I_{CM} = \frac{R+2H}{2(R+H)}\cdot MR^2$$

c. $F = \frac{M}{1+r/R}\left[a\left(1+\frac{I_{CM}}{MR^2}\right) + g\sin\vartheta\right]$

d. $F_A = \frac{r}{R}F - a\cdot\frac{I_{CM}}{R^2}$

dalle equazioni cardinali.

e. $F = 50.9$ N

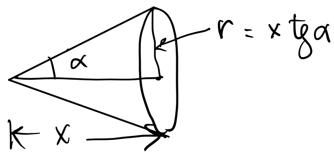
$F_A = 39.1$ N, rivolto verso l'alto lungo il piano inclinato.

f. $W_{ATTR} = 0$ (il punto di contatto è fermo), $P_{ATTR} = 0$

$$W_F = \Delta E_K - W_{Peso} = \frac{1}{2}Mv_F^2\left[1+\frac{I_{CM}}{MR^2}\right] + Mgd\sin\vartheta = 900\ \text{J}$$

$$P_F = \vec F\cdot\vec v + \vec \tau\cdot\vec \omega \Rightarrow P_{F_{max}} = F v_F\left(1+\frac{r}{R}\right) = 91.6\ \text{W}$$

3. Il volume del cono è $\frac{\text{base}\cdot\text{altezza}}{3} = \frac{x^2 \cdot tg^2\alpha \cdot x}{3} \propto x^3$



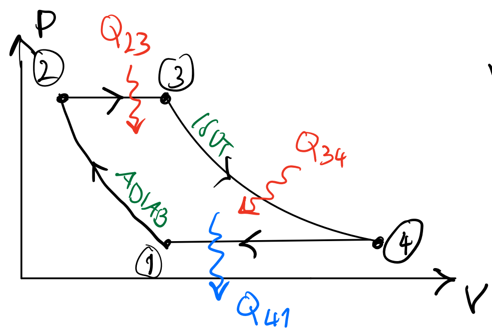
a. Nella trasformazione adiabatica $TV^{\gamma-1} = \text{costante} \Rightarrow \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}$, $\frac{V_1}{V_2} = \left(\frac{x_1}{x_2}\right)^3$

$$\Rightarrow \gamma = 1 + \frac{1}{3}\frac{\ln T_2/T_1}{\ln x_1/x_2} = 1.40 \sim \frac{7}{5} \Rightarrow \text{gas biatomico.}$$

b. lungo l'isobara $T_3 = T_2 \cdot \frac{V_3}{V_2} = T_2 \cdot \frac{V_1}{V_2} = T_2\left(\frac{x_1}{x_2}\right)^3 = 915\,K$

$$Q_{23} = n c_p (T_3 - T_2) = 1.05 \times 10^5\,J \Rightarrow n = \frac{Q_{23}}{c_p (T_3 - T_2)} = 6.8 \text{ (moli)}$$

c.



$$W_{TOT} = Q_{TOT} = \underbrace{Q_{23} + Q_{34}}_{\text{assorb.}} + \underbrace{Q_{41}}_{\text{ceduto}}$$

$$\eta = \frac{W}{Q_{assorb}} = 1 - \frac{|Q_{41}|}{Q_{23} + Q_{34}}$$

$$Q_{34} = nRT_3 \ln\frac{V_4}{V_3} = nRT_3 \ln\frac{V_1}{V_3} = nRT_3 \ln\frac{T_3}{T_1} = 6.25 \times 10^4\,J$$

$$Q_{41} = n c_p (T_1 - T_4) = n c_p (T_1 - T_3) = -1.27 \times 10^5\,J$$

$$\Rightarrow \eta = 0.24$$

d. $$\eta_{carnot} = 1 - \frac{T_{min}}{T_{MAX}} = 1 - \frac{T_1}{T_3} = 1 - \frac{273}{915} = 0.70 > 0.24$$

e. $\Delta S_{gas} = 0$, $\Delta S_{univ} = 0$ (trasf. reversibili) $\Rightarrow \Delta S_{amb.} = 0$

f. $\Delta S_{gas} = 0$, $\Delta S_{univ} > 0$ (trasf. irreversibile) $\Rightarrow \Delta S_{ambiente} > 0$