An Introduction to the New SI

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Plans are under way to redefine the International System of Units (SI) around 2018. The new SI specifies the values of certain physical constants to define units. This article explains the new SI in order to provide a resource for high school teachers as well as for advanced students already familiar with the pre-2018 SI.

Physical science is based on measurements, and the results of measurements are expressed in terms of units. For example, someone can measure the length of a table and report that it is 1.4 m long, where m stands for meters. This statement provides useful information to other people, because there is agreement about what a meter is. In fact, there is a treaty among 58 nations, including the United States, that codifies agreement on exactly what the meter is. The treaty is called the *Convention du Mètre* or Treaty of the Meter, and it also specifies other units.[1] According to the treaty, *Le Systèm International d'Unités* (SI), or the International System of Units, is agreed to be the standard by which all participating member states set their units. Even though a majority of people in the United States still use units such as inches and pounds, the official standards for these units are linked to the SI. For example, the U.S. definition of the inch is that it is exactly 0.0254 m or 2.54 cm, where the c stands for 10^{-2}.

Although the practice of establishing standards for measurements dates back thousands of years, the Treaty of the Meter was only established in 1875 with 17 nations initially signing on, including the United States. Incidentally, the anniversary of the signing, May 20th, is now known as World Metrology Day. The SI, established in 1960, is still more recent and continues to evolve.[2,3] In fact, it is anticipated that the definitions of measurement standards specified by the SI will undergo a substantial revision in 2018.[4-6] In the new SI, units are defined by assigning specific values to a set of physical constants.[7,8] Despite the fact that the definitions will be changed, the effect on everyday measurements will be imperceptible. This paper describes the new SI and shows how units will be based on assigned values of the constants.

I. A short history of length

The character of the new SI, in which the units are defined by defining the values of particular constants of nature when they are expressed in SI units, can be understood in part by recalling the history of the unit of length.

In antiquity the unit of length was often tied to the dimensions of the human body. A "foot" was the length of a foot, a "hand" was the width of a hand, a "cubit" was the length from the elbow to the fingertip, and so forth. The convenience of such an arrangement is obvious—the standard of length was always available and easy to use. The difficulty is also obvious—the standard was not really standard, varying with each individual. People tried various approaches to remedy this difficulty: The foot was the length of the king's foot or the average of the lengths of some number of people's feet. Such stratagems helped, but did not eliminate the problem.

The ancient Egyptians took an approach that is decidedly modern. Building the pyramids demanded precision and consistency in measurement. Ancient metrologists established a "primary standard" royal cubit in the form of a rod of a certain length (possibly originally set by the dimensions of the body of some pharaoh). Artisans who did the actual cutting and sizing of stones used a "working standard" of length that was compared regularly to the primary standard. In this way the Egyptian engineers guaranteed a consistent measure of length, protecting the primary standard from the wear and tear of everyday use, while providing easy access to the standard by transferring its accuracy to the working standards in the field.

Merchants of medieval Europe used a similar approach. Towns would embed a standard of length into a masonry wall in the town square, and all who did business in that town could confirm that goods sold by length were measured consistently. Variations from town to town could, however, cause problems. Other approaches seem even more modern: The inch was defined as the length of three barley corns. This connection to a universal feature of nature had an obvious appeal, but variations in weather from one growing season to the next, as well as variations among different plants, made such a standard less than perfect.

The French revolution brought an opportunity for both scientific and political reforms to the measurement of length. The size of the Earth was taken to be a standard that was both unchanging and universally available, so the meter was defined as a part in ten million of the distance from the equator to the pole, along the meridian through Paris. The difficulty of surveying even the accessible part of the pole-to-equator arc was great enough that the legal standard of length became the "meter of the archives," a platinum rod whose end-to-end length was close to the meter based on the surveyed distance.[9] Eventually the meter and the metric system as a whole spread beyond France and became more widely and internationally accepted. The 1875 Treaty of the Meter established the *Bureau International des Poids et Mesures* (BIPM) or International Bureau of Weights and Measures with the intention, among other things, of creating a new international standard of length, the International Prototype Meter (IPM). This was a platinum-iridium bar with fine lines (scratches) the distance between whose centers was one meter. This "line standard" had an obvious advantage over the earlier "end standard"

of the archives in that using it did not wear down its length. Copies of this bar, with calibrations of their differences from the IPM, were distributed to the signatories of the treaty.

Then the international community had a system for measuring length analogous to that of the ancient Egyptians. But problems soon arose. The distance between the scratches on meter bars could be compared with an accuracy on the order of one or two tenths of a micrometer, that is, about a part in 10^7. The ability to compare wavelengths of light and to compare distances measured in terms of such wavelengths outstripped the measurement of lengths based on the distance between two scratches on a metal bar. From early in the 20th century, metrologists juggled two systems of length measurement: the legal system based on the meter bar and a (more precise) de facto one based on the wavelength of light from a cadmium lamp.

This uncomfortable situation was rectified in 1960 when the international community agreed on a new definition of the legal (SI) meter based on the wavelength of light from a krypton lamp. In this way, the best metrological techniques of optics at the time were used for determining lengths. But the seeds of the destruction of even this advanced definition were already in the wind. The year 1960 saw the first demonstration of a laser, and it soon was clear that laser light could be more "pure" (with a narrower spread of wavelengths) than a krypton lamp could provide. Furthermore, the distribution of krypton wavelengths was found to be asymmetric, making the definition of the meter ambiguous. The result was that, as in the early 20th century, metrologists of the late 20th century began to use a de facto standard of length based on the wavelength of a helium-neon laser locked to the wavelength of a particular transition in the I_2 molecule. Many thought that a new definition would follow, based on the new de facto standard, as it had in 1960. But a better idea emerged, one that was a harbinger of the proposed 2018 reform of the SI.

In 1983, the international community adopted a definition of the meter as the distance that light travels in a certain length of time. Effectively, this defines the meter by defining the value of c, the speed of light, when expressed in the SI unit meters per second. One of the beauties of this definition, in contrast to the earlier definitions of the meter in terms of the cadmium or krypton wavelengths, is that it makes no choice of an atom or molecule as the standard. Anyone with a good laser, and a means of measuring the frequency f of the light it emits, automatically has a length standard graduated in wavelengths λ of light according to the expression $\lambda = c/f$. (See Secs. II a and II b for a more detailed explanation of the wavelength-frequency relation and how wavelength-scale lengths are related to meter-scale lengths.) Furthermore, the definition is based on a (presumably) unchanging and universal property of nature, the speed of light in vacuum. This beautiful definition makes the standard of length available to everyone at all times. If better lasers and better ways of measuring their frequencies become available, the same definition will embody those improvements. There will be no need to repeat the sad history of the 20th century's length definition, switching from Pt-Ir bars to Cd to Kr to I_2.

Today we are reforming the SI in a manner analogous to the beautiful definition of the meter.

II. The defining constants

We next explain how, by defining other constants of nature, we can define other units of measurement in the same way that defining the value of c defines the meter.

a. Frequency and time

The frequency standard for the SI is established by assigning a value to the frequency of a particular atomic transition associated with the energy difference between two discrete atomic energy levels. That frequency is given by the Planck relation $\Delta E = hf$, where ΔE is the energy difference, h is the Planck constant, and f is the frequency. It is currently, and for a while in the new SI will continue to be, the frequency corresponding to the transition between the two hyperfine levels in the ground state of a cesium atom. Here frequency refers to the frequency of the electromagnetic radiation absorbed or emitted when the atom makes a transition between the two hyperfine levels. The value assigned to the frequency is 9 192 631 770 cycles per second, where the unit cycles per second is called hertz or Hz. (In the fields of units and fundamental constants it is conventional to express long numbers with groups of three digits separated by small spaces rather than commas.) This value, about 9 GHz (the G stands for 10^9), is in the microwave frequency range, a few times higher than the frequency of kitchen microwave ovens. This transition frequency is one of the defining constants in the new SI.

Once this standard of frequency is defined, it is possible to determine other frequencies by comparison to this standard or to determine a second by counting cycles of the frequency. There are commercial instruments that can count periodic cycles at this frequency. Since there are exactly 9 192 631 770 cycles of the radiation in a second, we can time one second by counting off that number of cycles of the standard frequency. This is the principle behind atomic clocks.

b. Speed and length

Microwaves and light are forms of electromagnetic radiation that differ only in their frequencies. The frequency of visible light ranges roughly from 400 THz to 750 THz (the T stands for 10^{12}). Despite the difference in frequency, all electromagnetic radiation travels at the same speed in vacuum, even if it is measured by someone moving relative to the source of the radiation. This is the principle of Einstein's theory of special relativity. This speed is just the speed of light, denoted by c, which has an assigned value of $c = 299\ 792\ 458$ meters per second in both the current SI and the new SI. This provides a universal speed standard to which other speeds may be compared. The speed of light is one of the defining constants in the new SI.

At the same time, as described in Sec. II, the value of c

provides a standard for length in the SI. The wavelength of light, denoted by λ, is the distance between peaks of the electromagnetic radiation. For light traveling past a point in space, the number of wavelengths that go by in a second is its frequency in hertz. So the total distance traveled by light in one second is the wavelength times the number of wavelengths that pass a stationary point, or the wavelength times the frequency. That frequency, of course, is measured relative to the cesium hyperfine frequency discussed in Sec. II a. This distance traveled is just 299 792 458 m because the value of the speed of light is that many meters per second. This is expressed by the equation $\lambda f = c$. Because of this relationship, for light of a given frequency the wavelength will be a certain known length, which can be used to determine the length of one meter. In practice, one way of doing this is by using an interferometer, which allows distances to be determined by counting off wavelengths as the distance between two mirrors changes.

c. Voltage and resistance

Modern measurements of electrical quantities are based on two quantum phenomena from condensed matter physics, namely the Josephson effect and the quantum Hall effect.

The Josephson effect yields a standard of voltage equal to an applied frequency divided by the Josephson constant K_J (about 484 THz/V, where V is the symbol for the unit volt). Resistance measurements are made with the quantum Hall effect, which yields a specific ratio of voltage to current, called the von Klitzing constant R_K (about 25 813 Ω, where the unit is ohms). Since voltage and resistance can be measured in terms of these two constants, it follows from Ohm's law that current can also be determined by measuring the voltage across a resistor of known resistance when the current flows through.

Now, both the Josephson and von Klitzing constants are accurately, but not exactly, known. On the other hand, the measurements of voltage and resistance in terms of these constants can be made even more accurately than the constants themselves are presently known in SI units. In the new SI, these constants will be defined as described in the next section.

d. Charge and the energy-frequency relation

The elementary unit of electrical charge, denoted by e, is the charge of a proton or the negative of the charge of an electron. It is approximately $e = 1.6 \times 10^{-19}$ C, where C stands for coulombs. In the new SI, the elementary unit of charge e is assigned an exact value.

The Planck constant, denoted by h, is a universal constant associated with quantum mechanics and is approximately 6.6×10^{-34} J s. This constant has a number of roles in quantum physics. As seen in Sec. II a, it is the constant of proportionality that gives the energy of a photon, a quantum of electromagnetic radiation, in terms of its frequency f: $E = hf$. In this role, the unit for the Planck constant can be taken to be J/Hz.

The Planck constant is also the elementary unit of angular momentum; the smallest angular momentum of an elementary particle such as the electron is $h/4\pi$. The Planck constant also appears in the Heisenberg uncertainty relation. In the new SI, the Planck constant has an exact assigned value.

An additional role of the Planck constant is in the Josephson effect and the quantum Hall effect. Both the Josephson constant and the von Klitzing constant are simple functions of e and h: $K_J = 2e/h$ and $R_K = h/e^2$.

Since in the new SI, both the electric charge and the Planck constant are assigned exact values, this defines the units of charge C, voltage V, resistance Ω, and current A (ampere). As an additional consequence, because of the relation to the Josephson and von Klitzing constants, both of those constants will also be exactly known, and electrical measurements as described in the previous section will give results in terms of exactly known constants in the SI.

The assigned values of e and h that will be used for the redefinition are not yet completely set. The exact values will depend on the experimental information available at the time of the redefinition.

e. Mass and power

The kilogram is the SI unit for mass. In the new SI, the mass of an object in kilograms can be determined from the units already defined. Since energy can be expressed as $E = hf$ and, recalling Einstein's famous relationship $E = mc^2$, we see that the unit for mass must be related to the already defined units for h, c, and f, but this does not provide a practical means of defining the kilogram. Once the unit of charge and the Planck constant are defined, electrical units in terms of the Josephson and von Klitzing constants are known and, as described in the following, this is enough to define in a practical way the unit of mass.

The principle can be understood through a simple thought experiment to measure mass. Consider a perfect electric motor that can wind up a massless string that has a mass hanging at the end. If the motor is run to lift the mass, it will work against the force of gravity on the mass. The motor will do a certain amount of work, which is the increase in potential energy of the mass and is proportional to the mass. The same electrical energy is consumed by the ideal motor to lift the mass: voltage times the current being supplied to the motor multiplied by the time to lift the mass. Since these quantities can be measured with the electrical units already defined, the mass can be determined in the SI unit kilogram.

The local acceleration of gravity is also needed in order to know the force of gravity on the mass. It can be determined by dropping an object and measuring its acceleration with the time and speed units already defined.

In this way, comparing mechanical energy to electrical energy, mass can be measured. The most precise way of doing this is using an apparatus called the watt balance, where the watt is the unit of power, or energy per unit time. (In the future, the watt balance will be called the Kibble balance in rec-

ognition of the fact that it was invented by the physicist Brian Kibble.) The watt balance is essentially an electrical scale that measures mass by weighing it in the presence of gravity. To avoid friction, which would be a problem for the simple motor example mentioned, the watt balance makes the measurement in two steps. One step is the measurement of the current through a coil in a magnetic field that supports a mass with no motion, and the other step is the determination of the magnetic field by moving the coil through it and measuring the induced voltage on the coil. The result is a combination of electrical quantities that is equivalent to power, which is the source of the name of the apparatus. Model watt balances that can be constructed and demonstrated in the classroom are described by Quinn et al.[10] and Chao et al.[11]

Electrical power, the product $V I = V (V'/R)$, where the current is determined by measuring the voltage V' across a resistor with resistance R, and is proportional to the combination $K_J^2 R_K = 4/h$. The electric charge cancels from this combination, so it does not need to be exactly known in order to measure mass. The mass determination depends only on h and the time and length standards mentioned earlier. However, defining e also allows for separate measurement of both voltage and resistance.

Another method of determining mass in the new SI is to produce an object with a mass of one kilogram as a standard. This can accurately be done based on the fact that in the new SI, because the Planck constant is exact, the mass of one silicon-28 atom will be known in kilograms with a precision of about 10 significant figures from experiments that, for example, measure the recoil of an atom when it absorbs a photon of known frequency. A spherical single crystal of these atoms can be made with the suitable number of atoms needed to make up one kilogram. The number of atoms in the spherical crystal is determined by the measured volume of the sphere and the measured average spacing between the atoms in the crystal structure. This provides a method of producing an object with a mass of one kilogram that is independent of the watt balance.

Historically and currently, the kilogram is defined as the mass on an object, the International Prototype Kilogram, kept at the BIPM. The apparent instability of this mass, and the general discomfort of having an international system of units that relies on the existence of a single physical artifact, are major motivations to create the new SI.

f. Amount of substance

The amount of stuff or, more precisely, amount of substance in the SI is related to the number of items, atoms, molecules, etc. under consideration. The unit of amount of substance is the mole, which consists of approximately 6.02×10^{23} items. In the new SI, this number is fixed by assigning an exact value to the Avogadro constant, denoted by N_A, which is defined to be that number of items per mole. In the current SI, the mole is the number of atoms of ^{12}C in 12 g (0.012 kg) of ^{12}C. As in the case of the elementary charge and the Planck constant, the exact value of N_A in the new SI will be determined by the results of experiments done up to the time of the redefinition to insure the continuity of the definition of the mole.

g. Temperature

In the new SI, the Boltzmann constant is one of the exactly defined constants, so temperature can be measured, for example, with a gas thermometer based on the ideal gas law, $pv = nRT$. In this equation, p is the pressure of the gas in a container, v is the volume of the container, n is the number of moles of gas in the container, R is the molar gas constant, and T is the temperature of the gas. One can imagine that the gas container is in contact with the object whose temperature is being measured, so the temperature of the gas will eventually be equal to the temperature to be measured. The temperature T of the gas may be found by determining the rest of the terms in the ideal gas law equation and solving for T.

This can be done, based on the definitions already mentioned, as follows. The pressure p is force per unit area. A standard for force can be calibrated by comparison to the force of gravity on a known mass, and area is just length squared. The volume v is just length cubed. The number of moles n of gas in the chamber is the total number of atoms N divided by the Avogadro constant, $n = N/N_A$. This leaves the problem of determining the molar gas constant R, which is related to the Boltzmann constant k by $R = N_A k$. Because the Boltzmann constant and N_A are exactly defined in the new SI, that means that R is also known exactly, by definition. So, using this definition, we get the temperature from the various other measured aspects of the gas thermometer.

Note that from the fact that $nR = Nk$, it can be seen that the Avogadro constant is not necessary for such a measurement of temperature, although it provides a more convenient measure of the amount of gas than the number of atoms does.

The Boltzmann constant is approximately 1.38×10^{-23} J/K, where K or kelvin is the unit of temperature in the SI. Evidently, the product kT has units of energy. In fact, other methods of measuring temperature also involve measuring energy and the combination kT is what is measured. In fact, kT is proportional to the average kinetic energy of an atom in the ideal gas. In this sense, the Boltzmann constant is a conversion factor from energy to temperature.

h. Illumination

For illumination, the candela, abbreviated cd, is the unit of luminous intensity, which is the power per solid angle emitted by a light source in a particular direction multiplied by a factor K_{cd}, which takes into account the sensitivity of the eye to various colors of light. One candela is approximately the luminous intensity observed for a candle, from which it gets its name.

If a source of light has a luminous intensity of one candela in every direction, then its total luminous flux is 4π lumens. In the new SI, as in the current SI, the candela is defined by

specifying that the factor K_{cd} for green light at 540 THz is exactly 683 lumen per watt.

III. New SI

The previous sections should make it clear how the specification of the values of seven constants defines the new SI units. This is the basis for the new SI, which together with the definitions of the various units with special names, such as the joule, in terms of the units defined by the constants, is sufficient to define the entire SI. The International Committee for Weights and Measures intends to propose a revision of the SI in 2018 as follows:

Efffective May 20, 2019, the International System of Units is the system of units in which:

- the unperturbed ground state hyperfine transition frequency of the cesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 Hz,
- the speed of light in vacuum c is 299 792 458 m/s,
- the Planck constant h is $6.626\,070\ldots\times10^{-34}$ J s,
- the elementary charge e is $1.602\,176\,6\ldots\times10^{-19}$ C,
- the Boltzmann constant k is $1.380\,64\ldots\times10^{-23}$ J/K,
- the Avogadro constant N_A is $6.022\,140\ldots\times10^{23}$ mol^{-1},
- the luminous efficacy K_{cd} of monochromatic radiation of frequency 540×10^{12} Hz is 683 lm/W,

where the dots (. . .) will be replaced by additional digits some of which have yet to be determined by experiments to be done up to the time of the redefinition. From then on, the constants will be defined to be exactly those values.

IV. New SI vs. current SI

The essential change from the current SI to the new SI is in the definitions of the units.

In the current SI, seven base units are defined by the following statements:

- The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.
- The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
- The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
- The ampere is that constant current that, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per meter of length.
- The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

- *1.* The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is "mol."

 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

- The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.

In the new SI, no units need to be singled out as base units. The concept of base units is a carryover from when the meter and kilogram were both defined in terms of metal objects. In the new SI, it is recognized that the definitions need not be directly linked to the traditional base units in a one-to-one relationship. For example, in the new SI, the speed of light c is given a particular value and that defines the combination of units m/s. Similarly, giving h a particular value defines the combination J s or kg m^2/s or J/Hz. The defining constants specify combinations of units such that each of the traditional base units is unambiguously determined.[12]

Although the forms of all of the definitions are different, some of the units are unchanged in the new SI. In particular, the meter, second, and candela are the same in the new and current SI. On the other hand, the kilogram, ampere, kelvin, and mole are defined differently. In fact, in the current SI, the kilogram is still the mass of a metal artifact kept in a vault near Paris, as it has been since 1889.

V. Why change?

The new SI is a significant improvement over the current SI, which is the reason for changing. Some of the improvements are as follows.

The current definition of the kilogram, in terms of a metal artifact, is problematic because the mass of the artifact is changing relative to the mass of similar copies. In addition, it needs to be washed before being used for measurements, and in that process the mass also changes. These variations are larger than the uncertainty in the measurement of mass using a watt balance or a silicon crystal sphere for a comparison standard, so the new SI will allow for more reliable mass measurements.

Electrical measurements using the Josephson effect and the quantum Hall effect are more accurate than the Josephson and von Klitzing constant are currently known in terms of SI units. As a result, since 1990, electrical measurements have been made in terms of assigned values for these constants. The assigned values are not directly tied to the SI, so electrical measurements are not actually done in terms of SI units and are slightly different. Moreover, the conventional electrical units are the legal units in the United States and most other countries, meaning, for example, that we pay for electrical energy measured in conventional units, not in the SI. With the exact specification of the electric charge and Planck constant

in the new SI, the Josephson and von Klitzing constants are well defined, and the results of electrical measurements will be given in actual SI units.

The new definition of the mole specifies the number of entities in a mole. In the current SI, the mole is the number of carbon atoms in 0.012 kg of carbon, but the actual number is not given, which is less clear.

For temperature, the kelvin is currently defined in terms of the triple point of water, that is, the unique temperature at which water coexists as ice, liquid, and vapor. However, this property of water depends on the purity and isotopic composition of the water used. The kelvin is better defined if linked to an exact numerical value of the Boltzmann constant k.

When expressed in terms of the units of the new SI, the values of many physical constants have smaller uncertainties, and some, in addition to the defining constants, are exact.

Beyond that, the new SI achieves a dream that was expressed as early as the 19th century by J. C. Maxwell, one of the greatest scientific visionaries in history. In considering the use of the dimensions of the Earth and its rate of rotation as standards of length and time, Maxwell recognizes their possible variability and concludes, "If, then we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and absolute mass of these imperishable and unalterable and perfectly similar molecules."

Terry Quinn, the prominent director emeritus of the *Bureau International des Poids et Mesures* (BIPM), quotes Maxwell's commentary and observes, "At the time, science and technology were not sufficiently advanced for Maxwell's precepts to be implemented. However, at the beginning of the 21st century, they are, and we are on the brink of implementing Maxwell's precepts for a system of units based not simply on the properties of 'these imperishable and unalterable and perfectly similar molecules' but on what we call these days the fundamental constants of physics."[7-9] Today, Maxwell's dream and Quinn's prediction are being realized.

VI. Topics for further study

Some topics related to the redefinition that could be the basis for further studies are:[7-11]

- History of units.
- Determination of physical constants.
- Hyperfine splitting frequency.
- Interferometry.
- Josephson effect.
- Quantum Hall effect.
- Watt balance experiment.
- Crystal lattice spacing measurements.
- Atom recoil experiments.
- Temperature measurements.

VII. Web resources

Websites that provide further information are:

- The SI Redefinition Portal, U.S. National Institute of Standards and Technology, http://nist.gov/pml/si-redef/index.cfm
- Measurement Units, International Bureau of Weights and Measures, http://SIbipm.org/en/measurement-units

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What Exactly Are the New Definitions of Kilogram and Other SI Units?

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On November 16, 2018, at the 26th meeting of the General Conference on Weights and Measures (CGPM), a resolution was adopted that introduced historical changes into the International System of Units (SI). This decision, effective from May 20, 2019, established that the SI units of measurement will be defined by means of a set of seven fixed numerical values of natural constants.[1-7] The kilogram, the basic unit of mass, is no longer defined by a standard prototype, but relies on the Planck constant h. The definitions of the base SI units ampere, kelvin, and mole will also change. This will accomplish the long-term dream of scientists who have been claiming that measurement systems should be based on natural units.[4,5] In this paper we derive the natural units as combinations of the seven defining natural constants and determine the numerical relations between the natural units and the units of SI.

The new definitions of the SI units

There is plenty of literature dedicated to the history of SI, and the earlier definitions of the base SI units can be found in many publications. Here we focus our attention to the new definitions[1-4] and analyze their meaning. The new definitions of the seven base units are shown in Table I. Each of them is characterized by one of the seven natural constants. The first constant is the caesium frequency Δv_{Cs}, well known as the measurement standard for precise atomic clocks.[8] Next are the five universal constants c, h, e, k_B, and N_A, which are involved in the physical laws in classical and quantum physics. The last constant K_{cd} is a conveniently chosen fixed numerical value of luminous efficacy.[9]

The new definitions in Table I are instructive and complete. They are, however, very concise; therefore, the practical guidance is provided on the BIPM website,[7] as mentioned in Ref. 6. From them one concludes that the definitions of the second, the meter, and the candela have not changed, although they are now written differently. The change in the definition of the watt has no practical consequences for the candela.

There remain the four important base units, including the kilogram, whose definitions we analyze in the following section. We describe the mathematical procedure to obtain for each of them both the natural unit and the factor connecting this natural unit to the corresponding SI unit.

Seven natural constants – Seven equations!

When the seven constants are written together (Table II), one immediately observes that these are actually a system of seven algebraic equations, in which the units s, m, kg, A, K, mol, and cd are the unknown variables.[10-12] It is obvious that by solving these equations, one obtains expressions containing the universal natural units and the numerical factors for each base unit of the SI system. It should be stressed again that the seven defining constants are fixed, i.e., their numerical values are exact.

The new definition of kilogram and other base units of SI

Solving the first equation in Table II gives the definition of the second:

$$1\,\mathrm{s} = \left(\frac{1}{\Delta v_{Cs}}\right) \cdot 9192631770. \tag{1}$$

Combined with the second row in Table II, this gives the solution for the meter:

$$1\,\mathrm{m} = \left(\frac{c}{299792458}\right)\mathrm{s} = \frac{c}{299792458} \times \frac{91912631770}{\Delta v_{Cs}}, \tag{2}$$

$$1\,\mathrm{m} = \left(\frac{c}{\Delta v_{Cs}}\right) \times \frac{9192631770}{299792458} = \left(\frac{c}{\Delta v_{Cs}}\right) \cdot 30.66331899. \tag{3}$$

These are the new definitions of the second and the meter, whereas the expressions

$$\left(\frac{1}{\Delta v_{Cs}}\right) \text{ and } \left(\frac{c}{\Delta v_{Cs}}\right)$$

are the natural units for time and length, respectively.

By solving the third row in Table II and using the solution for the second and the meter, one obtains the new definition of kilogram[10]:

$$1\,\mathrm{kg} = \left(\frac{h\Delta v_{Cs}}{c^2}\right) \cdot 1.475521400 \times 10^{40}. \tag{4}$$

The expression $\left(\dfrac{h\Delta v_{Cs}}{c^2}\right)$ is the natural unit for mass. If this equation is multiplied by c^2, one obtains

$$1\,\mathrm{kg} \cdot c^2 = (h\Delta v_{Cs}) \cdot 1.475521400 \times 10^{40}. \tag{5}$$

This leads to a physical interpretation of the kilogram, which is:

> A body has a mass of 1 kg if its rest energy is equal to the energy of 1.4755214×10^{40} caesium photons.

This shows that the obtained numerical factors also have a physical meaning and are important parts of the unit definitions. (It should be said, however, that such an interpretation does not suggest a practical way to implement the definition.) Continuing with the similar procedure, one solves all seven equations and obtains Table III, where each of the seven base units is expressed as a natural unit (which is a combination of the seven natural constants), multiplied by a calculated numerical factor.

Table I. New definitions of the seven base units of SI.

SECOND	The second, symbol s, is the SI unit of <u>time</u>. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition <u>frequency of the caesium-133 atom</u>, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.
METER	The meter, symbol m, is the SI unit of <u>length</u>. It is defined by taking the fixed numerical value of the <u>speed of light</u> in vacuum c to be 299792458 when expressed in the unit $m \cdot s^{-1}$, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.
KILOGRAM	The kilogram, symbol kg, is the SI unit of <u>mass</u>. It is defined by taking the fixed numerical value of the <u>Planck constant</u> h to be 6.62607015×10^{-34} when expressed in the unit J·s, which is equal to $kg \cdot m^2 \cdot s^{-1}$, where the meter and the second are defined in terms of c and $\Delta\nu_{Cs}$.
AMPERE	The ampere, symbol A, is the SI unit of <u>electric current</u>. It is defined by taking the fixed numerical value of the <u>elementary charge</u> e to be $1.602176634\times10^{-19}$ when expressed in the unit C, which is equal to A·s, where the second is defined in terms of $\Delta\nu_{Cs}$.
KELVIN	The kelvin, symbol K, is the SI unit of <u>thermodynamic temperature</u>. It is defined by taking the fixed numerical value of the <u>Boltzmann constant</u> k_B to be 1.380649×10^{-23} when expressed in the unit $J \cdot K^{-1}$, which is equal to $kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}$, where the kilogram, meter, and second are defined in terms of h, c, and $\Delta\nu_{Cs}$.
MOLE	The mole, symbol mol, is the SI unit of <u>amount of substance</u>. One mole contains exactly 6.02214076×10^{23} elementary entities. This number is the fixed numerical value of the <u>Avogadro constant</u>, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
CANDELA	The candela, symbol cd, is the SI unit of <u>luminous intensity</u> in a given direction. It is defined by taking the fixed numerical value of the <u>luminous efficacy</u> of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit $lm \cdot W^{-1}$, which is equal to $cd \cdot sr \cdot W^{-1}$, or $cd \cdot sr \cdot kg^{-1} \cdot m^{-2} \cdot s^3$, where the kilogram, meter, and second are defined in terms of h, c, and $\Delta\nu_{Cs}$.

Relations between the natural units and the derived units of SI

The method used for the base units and the obtained results shown in Table III are applied to calculations of the corresponding relations for units of the derived physical quantities. The results for a number of derived units are presented in Table IV.

Several important conclusions follow from the analysis of Tables III and IV. The natural units for the base and derived quantities are obtained by solving equations of Table II. They are combinations of the natural constants, with the correct dimension, and no additional dimensional analysis is necessary. It should be pointed out that now there is no essential distinction between the base

Table II. Natural constants involved in the new definitions of the SI base units.

The value of the natural constants expressed in the SI base units	
$\Delta\nu_{Cs} =$	9192631770 s^{-1}
$c =$	299792458 $m \cdot s^{-1}$
$h =$	6.62607015×10^{-34} $kg \cdot m^2 \cdot s^{-1}$
$e =$	$1.602176634 \times 10^{-19}$ A·s
$k_B =$	1.380649×10^{-23} $kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}$
$N_A =$	$6.02214076 \times 10^{23}$ mol^{-1}
$K_{cd} =$	683 $cd \cdot sr \cdot kg^{-1} \cdot m^{-2} \cdot s^3$

and the derived units. In principle, all SI units can be derived from other sets of seven basic units. (For example, in the revised SI the joule could have replaced the kilogram and the ohm could have replaced the ampere as basic units, with no change needed to the seven defining constants.) This is stressed by the fact that some of the natural units are extremely simple, containing only one or two constants. These are the units for the frequency ($\Delta\nu_{Cs}$), velocity (c), action (h), or electric charge (e), as well as those for the magnetic flux (h/e) and the electric conductance (e^2/h). Furthermore, in the natural units we recognize the elements of some fundamental equations for the energy ($E = h\nu$, $E = mc^2$, $E = k_B T$, and $E = eU$, where ν, m, T, and U denote the frequency, mass, temperature and voltage, respectively), appearing in classical, quantum, relativistic, statistical, and atomic physics.

Educational benefits from the new definitions

Teachers of introductory physics in high schools and colleges have now a great responsibility, since they will be the first who will have to explain to students and to the general public the importance of the new definitions of SI units. As stressed in Ref. 5, the new definitions offer many topics for further study: among others, the history of SI and experimental methods developed for obtaining accurate values of the natural constants[13] or the experimental setups to be used in the classroom,[14] for which, however, the absolute precision can not be expected. The literature recommended to teachers and students should be completed with the 9th edition of the SI brochure,[6] which describes the revised SI and is published online on the BIPM website.[7]

To these subjects we add the investigations of the natural units by solving appropriate equations and calculations of their numerical relationships. Their numerical values can be obtained by using the pocket calculator, but when high precision values are needed, the best solution is a computer code. (Even then the obtained numbers are only approximations but, if needed, the additional digits can always be calculated using the described method.) The program code is simple and can be realized with any of the computer languages accessible to students. The exponents of the seven constants contained in a natural unit are read as input. We did our calculations with codes in Python and FORTRAN. In the appendix[15] our Python code is listed, with the necessary explanations.

At the end, we conclude that the specific choice of a set of universal constants and properties of the resulting natural

Table III. Numerical relations between the natural units and the base units of SI.

Base quantity	Base unit of SI	Relations between the base units of SI and the natural units
time	second	$1\ \text{s} = (1/\Delta\nu_{Cs}) \cdot 9.192631770 \times 10^9$
length	meter	$1\ \text{m} = (c/\Delta\nu_{Cs}) \cdot 3.066331899 \times 10^1$
mass	kilogram	$1\ \text{kg} = (h\Delta\nu_{Cs}/c^2) \cdot 1.475521400 \times 10^{40}$
electric current	ampere	$1\ \text{A} = (e\Delta\nu_{Cs}) \cdot 6.789686817 \times 10^8$
thermodynamic temperature	kelvin	$1\ \text{K} = (h\Delta\nu_{Cs}/k_B) \cdot 2.266665265 \times 10^0$
amount of substance	mole	$1\ \text{mol} = (1/N_A) \cdot 6.022140760 \times 10^{23}$
luminous intensity	candela	$1\ \text{cd} = [K_{cd}\ h(\Delta\nu_{Cs})^2] \cdot 2.614830482 \times 10^{10}/\text{sr}$

Table IV. Numerical relations between the natural units and the derived SI units.

Derived quantity	Derived unit of SI	Relations between the derived units of SI and the natural units
velocity, speed	meter per second	$1\ \text{m·s}^{-1} = (c) \cdot 3.335640952 \times 10^{-9}$
acceleration	meter per second squared	$1\ \text{m·s}^{-2} = (c\,\Delta\nu_{Cs}) \cdot 3.628602815 \times 10^{-19}$
momentum	kilogram × meter per second	$\text{kg·m·s}^{-1} = (h\Delta\nu_{Cs}/c) \cdot 4.921809606 \times 10^{31}$
angular momentum	kilogram × meter squared per second	$1\ \text{kg·m}^2\text{·s}^{-1} = (h) \cdot 1.509190180 \times 10^{33}$
frequency	hertz	$1\ \text{Hz} = (\Delta\nu_{Cs}) \cdot 1.087827757 \times 10^{-10}$
energy, work	joule	$1\ \text{J} = (h\,\Delta\nu_{Cs}) \cdot 1.641738968 \times 10^{23}$
power	watt	$1\ \text{W} = h\,(\Delta\nu_{Cs})^2 \cdot 1.785929219 \times 10^{13}$
force	newton	$1\ \text{N} = [h\,(\Delta\nu_{Cs})^2/c] \cdot 5.354081105 \times 10^{21}$
pressure	pascal	$1\ \text{Pa} = [h\,(\Delta\nu_{Cs})^4/c^3] \cdot 5.694382340 \times 10^{18}$
action	joule × second	$1\ \text{J·s} = (h) \cdot 1.509190180 \times 10^{33}$
electric charge	coulomb	$1\ \text{C} = (e) \cdot 6.241509074 \times 10^{18}$
electric field	newton per coulomb	$1\ \text{N·C}^{-1} = [h(\Delta\nu_{Cs})^2/(e\,c)] \cdot 8.578183643 \times 10^2$
voltage	volt	$1\ \text{V} = (h\,\Delta\nu_{Cs}/e) \cdot 2.630355814 \times 10^4$
electric resistance	ohm	$1\ \Omega = (h/e^2) \cdot 3.874045865 \times 10^{-5}$
electric capacitance	farad	$1\ \text{F} = [e^2/(h\,\Delta\nu_{Cs})] \cdot 2.372876339 \times 10^{14}$
magnetic flux	weber	$1\ \text{Wb} = (h/e) \cdot 2.417989242 \times 10^{14}$
magnetic induction	tesla	$1\ \text{T} = [(h\,(\Delta\nu_{Cs})^2/(e\,c^2)] \cdot 2.571674759 \times 10^{11}$
electric conductance	siemens	$1\ \text{S} = (e^2/h) \cdot 2.581280746 \times 10^4$
inductivity	henry	$1\ \text{H} = [h/(\Delta\nu_{Cs}\,e^2)] \cdot 3.561267710 \times 10^5$
luminous flux	lumen	$1\ \text{lm} = [h(\Delta\nu_{Cs})^2\,K_{cd}] \cdot 2.614830482 \times 10^{10}$
illuminance	lux	$1\ \text{lx} = [h\,(\Delta\nu_{Cs})^4\,K_{cd}/c^2] \cdot 2.781027076 \times 10^7$
activity	becquerel	$1\ \text{Bq} = (\Delta\nu_{Cs}) \cdot 1.087827757 \times 10^{-10}$
absorbed dose	gray	$1\ \text{Gy} = (c^2) \cdot 1.112650056 \times 10^{-17}$
dose equivalent	sievert	$1\ \text{Sv} = (c^2) \cdot 1.112650056 \times 10^{-17}$

methods with the laws of physics, confirm that the redefinition of the SI units has a huge educational potential.

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15. Readers can view the appendix at *TPT Online,* http://dx.doi.org/10.1119/1.5141976 under the Supplemental tab.

units can lead to inspiring arguments in the discussions with students. The scaling factors obtained for various quantities may differ by 58 orders of magnitude, and this can be another creative idea for discussion. A wide choice of possible topics for discussion, connecting the measurement principles and

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